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TRANSLATION

OPTIMUM GEOMETRICAL RATIOS IN INDUCTION PUMPS FOR LIQUID METALS

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Optimum Geometrical Ratios in Induction Pumps for Liquid Metals

by

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Problem. The basic problem originating during the planning of induction pumps of all types, is the finding of their main dimensions and parameters height of channel 2b, its width 2a, pole graduation τ, slip s and active length of inductor l (fig. 1) To determine the basic dimensions of a pump at given pressured p_n and delivery Q requires the solution of the problem of optimum construction. Of most practical value is the finding of an optimum by the maximum of the hydraulic efficiency because induction pumps have a relatively low efficiency.

In spite of the fact that problems concerning optimum construction of induction pumps by the conditions of maximum efficiency have been examined by a number of authors, this circle of problems still cannot be considered as totally solved. Investigated were only special cases of optimum ratios at oversimplified conditions, and the initial tial conditions were sometimes disputable.

And so in the experiments by D.A.Watt (lit.1-3) optimum ratios were obtained from the minimum loss condition in the channel of the pump. The losses in inductor windings are disregarded, these losses amount to 30-50% of all losses. In addition, the use of the ratios listed in these reports is possible only after selected the height of the channel 2b. The latter to a large extent stipulates the magnitude of the non-magnetic gap and predetermines the characteristics and other dimensions of the pump. Finding the height of the channel should be the subject of the investigation.

This deficiency is endured also by certain other reports (lit.4-6). Due to the extreme simplification of the problem of L.G.Savvin a false conclusion is made with

respect to the ratio $\frac{b}{a}$ which should be close to unity [lit.7]. In real constructions it equals 0.115 - 0.033.

Optimum slipping corresponding to minimum n.s. at given pressure, found in the report of I.A.Tyutin and E.K.Yankop [4], formulas (45), (47), requires preselection of either channel height, or its width and pole graduation. According to the concept of the authors, the n.s. = no power minimum condition corresponds to maximum efficiency. But this assumption actually does not take place.

The mentioned experiments do to a known extent elucidate the problem, but the formulas derived in them are little suitable for the determination of main dimensions of the basic structural variant of induction pump construction. This brings up the need for newly examining the problem concerning optimum geometric ratios in induction pumps, on the basis of maximum efficiency conditions.

In practice of planning induction pumps is encountered the need of creating structures of minimum weight and overall dimensions, possesing together with satisfactory operational indices - efficiency and power coefficient. It is therefore important to have also another problem - selection of optimum geometric ratios, assuring minimum weight of active materials.

In this report are discussed two problems concerning the selection of optimum geometric ratios in induction pumps: judging by the maximum efficiency conditions and minimum weight of active materials. The principal trend of the first one of these was mentioned by A.I.Vol'dek [8]. The solution is applied to a plain pump with bilateral inductor, but the results are also applicable to cylindrical and spiral pumps [9].

<u>Basic conditions and assumptions.</u> We will consider as given: pressure p_n, delivery of pump Q, frequency f, parameters of pumping metal - temperature, electric conductivity σ, mass density ρ and kinematic viscosity v. On the basis of the properties of the metal and its temperature, is selected channel wall thickness b_k and thickness of heat insulation b_i. They are preselected or considered as fixed:

mean rate of flow of the metal v, temperature of induction winding, its linear current load A and current density in this winding j.

When planning an induction pump of minimum weight it is necessary to select such values of magnetic induction in the yoke B_{mj}, and in the teeth (and in the core in case of a cylindrical pump). The rate of flow of the metal should be taken maximum possible by the conditions of wear resistance of channel walls.

We will make the following assumptions:

1. We consider only the main forms of losses - in liquid metal, inductor windings and channel walls, constituting approximately 90% of all losses. Losses in steel, short circuiting busbars and hydraulic losses, which ordinarily do not exceed 10% of the total losses, are disregarded.

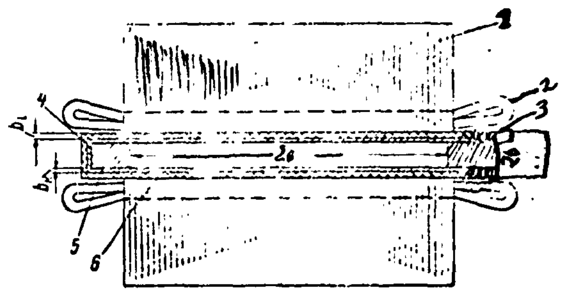

Fig.1.Schematic drawing of induction pump
1-inductor; 2-winding; 3-thermoinsulation; 4-channel; 5-copper busbars; 6-liquid metal.

2. We assume, that in plane pumps there are shorted side busbars (fig.1). Consequently the low influence of lateral boundary effect on the lowering of pressure and losses are considered a constant coefficient.

3. The longitudinal marginal effect is considered as empirical coefficients, which are assumed to be constant in the zone of change of variable values.

4. We disregard the influence of surface effect on pressure and losses.

5. The resultant coefficient of air gap and its components, e.g. coefficient of serration (Carter) etc. is also considered as constant at changes of nonmagnetic gap 2Δ and pole graduation.

Since the above mentioned coefficients are not strictly constant, then into the calculation should be introduced some of their mean values with possible subsequent improvement of same.

Hydraulic losses in induction pumps ordinarily do not exceed 2-8% of all losses. Figuring of same, is little reflected on the final results, and it leads to highly complex dependences, which can hardly be used in practice. On the other hand, selecting the rate of metal flow to a known extent predetermines the magnitude of hydraulic losses. Consequently they are not figured in when searching for optimum dimensions of pumps.

Optimum ratios by maximum efficiency conditions. In appendix 1 is given an equation of relative losses in an induction pump:

$$F(b, s) = \frac{\Sigma P}{P_2} = k_o k'_o \left\{ \frac{s}{1-s} + \frac{n}{b(1-s)s} + \frac{1}{c} \frac{[b + g_1(1-s)][\Delta^2(1-s)^4 + K^2(bs+n)^2]}{b(1-s)^2 s} \right\}. \qquad (1)$$

The relative losses appear to be the function of two arguments-slipping s and semi height of the channel b. All remaining coefficients in equation (1) values of which are given in appendix 1, can be considered as constant at changes in s and b. The optimum values s_η and b_η, corresponding to minimum relative losses, can be found by solving the system of equations:

$$\left. \begin{aligned} F'_b &= \frac{\partial}{\partial b}\left(\frac{\Sigma P}{P_2}\right) = 0; \\ F'_s &= \frac{\partial}{\partial s}\left(\frac{\Sigma P}{P_2}\right) = 0, \end{aligned} \right\} \qquad (1a)$$

which in inverted form can be presented as:

$$\begin{aligned} F'_b = {} & 2[K^2 s^3 + (1-s)^4] b^3 + [2nK^2 s + \\ & + g_1 K^2 (1-s) s^2 + 2d(1-s)^4 + \\ & + g_1 (1-s)^5] b^2 - [(cn + g_1 n^2 K^2)(1-s) + \\ & + g_1 d^2 (1-s)^5] = 0; \end{aligned} \qquad (2)$$

$$\begin{aligned} F'_s = {} & (1-s)[K^2 s^2 + (1-s)^4] b^3 + [4nK^2 s^2 + \\ & + g_1 K^2 (1-s) s^2 - 2d(1+s)(1-s)^4 - \\ & - g_1 (1+2s)(1-s)^5] b^2 + [(c + 2g_1 n K^2)(1-s) s^2 - \\ & - n^2 K^2 (1-3s) - d^2 (1+s)(1-s)^4 - \\ & - 2g_1 d (1+2s)(1-s)^5] b - \\ & - [(cn + g_1 n^2 K^2)(1-2s)(1-s) + \\ & + g_1 d^2 (1+2s)(1-s)^5] = 0. \end{aligned} \qquad (3)$$

These equations are valid for $b > 0$ and $1 \geq s \geq 0$. The system of equations (2),(3) can be solved graphically in the following manner. Dealing in values s we find the roots of the cubical equation (2) and we plot a graph $b_\eta = f_1(s)$. The olution of equation (3) for the very same s values will give a graph $s_\eta = f_2(b)$. The points of intersection of these curves determine optimum slipping values s_η and the half-heights of the channel b_η.

The coefficients of equation (2) for all mentioned s values retain positive values. Consequently in accordance with the Descartes theorem this equation will have only one positive root. The function $b_\eta = f_1(s)$ appears to be single valued, sufficiently smooth and limited. Equation (3) at $1 \geq s \geq 0$ may have a different number of sign changes in the series of polynomial F_s' coefficients depending upon the numerical values c,gl,n,d and K. Consequently the graph $s_\eta = f_2(b)$ can have one or two branches for $b > 0$. Practically the solution is obtained as single valued, because ordinarily in case of two branches of curve $s_\eta = f_2(b)$ they are both at the point of intersection with curve $b_\eta = f(s)$ fusing together into one.

In fig.2 in role of an example are shown graphs b_η and s_η plotted according to equations (2) and (3) for the IN-7 pump designed by the Physics Inst. of the Academy of Sciences of Latvian SSR for sodium at 400°C and a pressure of 5 kg/cm^2 and with a delivery of 250 m^3/hr [10]. The parameters and dimensions of this pump in plan were selected, on the basis of requirements to obtain maximum efficiency. In the plan were selected: $s = 0.25$, $b = 8$ mm. When solving the system of equations (2) and (3) were used the very same basic values and coefficients which have been adopted in [10] . From fig.2 we find $s_\eta = 0.216$, $b_\eta = 5.7$ mm. These numbers are perfectly suitable to be applied in role of basic calculation variant. The fact is if we should contemplate a reduction in the semi-height of the channel from 8 to 5.7 mm then this would lead to a slight rise in hydraulic losses which is essentially balanced by the reduction in losses in the copper of the inductors.

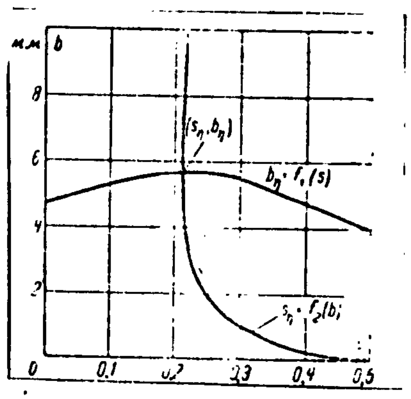

Fig. 2. Graphs $b_\eta = f_1(s)$ and $s_\eta = f_2(b)$ for IN-7 induction type pump designed for maximum efficiency for pumping sodium. $s_k = 0.268$; $s_\eta = 0.216$; $b_\eta = 5.7$ mm

Plotting of curves $b_\eta = f_1(s)$ and $s_\eta = f_2(b)$ requires the solution of a greater number of cubic equations and is connected with considerable time losses. For every day project operation more economical solutions are necessary. It appears, that it is possible to point toward a value of critical slipping s_k such, where $s_\eta \approx s_k$, and then by one-two approximations to find b_η and s_η.

We shall discuss the coefficient c_0 at a higher degree of the unknown in equation (3):

$$c_0 = (1+s)[K^2s^2 - (1-s)^4]. \qquad (4)$$

When c_0 tends toward zero one of the roots of equation (3) departs into infinity, because this equation transforms from cubical into square. It means that the steep sections of the curve $s_\eta = f_2(b)$, departing into infinity and obligatorily intersecting the graph with a limited function $b_\eta = f_1(s)$, will be situated near s_k, at which c_0 tends toward zero. Equating expression (4) to zero, we will determine the values of critical slipping and will take from them only this, which is situated in the zone $0 < s < 1$:

$$s_k \approx 1 + \frac{K}{2} - \sqrt{K + \frac{K^2}{4}}, \qquad (5)$$

where K - dimensionless parameter of pump:

$$K = \frac{\mu_0 \sigma v^2}{k_\delta \omega}; \qquad 5a$$

here ω - angular frequency; μ_0 - magnetic permeability of vacuum; k_δ - resultant coefficient of nonmagnetic gap.

The parameter K can be expressed also through the magnetic Re number $R_m = \mu_0 \sigma vb$ and relative frequency $\tilde{\omega} = \mu_0 \sigma \omega b^2$;

$$K=\frac{R_m^2}{k_1\bar{\omega}}. \quad 5\text{a}$$

Knowing s_k, the optimum values b_η and s_η can be found by the method of subsequent approximations. To determine the first approximation b_1 we substitute the value s_k in equation (2). Its positive root equals b_1. We will rewrite equation (3), arranging its members by degrees s:

$$a_6s^6+a_5s^5+a_4s^4+a_3s^3+a_2s^2+a_1s+a_0=0. \quad (6)$$

The coefficients of this equation appear to functions b. Their values are given in appendix II. Having substituted b_1 in equation (6) and having solved it relative to slipping, we find the first approximation s_1.

Equation (6) generally has six roots. A part of these roots appears to be negative, the others complex. For us it is sufficient to find only one positive root. Determination and improvement of root s_1 is made considerably easier, if we keep in mind, that s_1 is close to s_k (ordinarily $s_1 < s_k$, and the Horner table is used in combination with the method of linear interpolation [11]. By the value s_1 we find in an analogous method from equation (2) the second approximation b_2, and by it from equation (6) s_2. We could limit ourselves to the second approximation or continue with further improvement, but it is much simpler to determine the values b_η and s_η graphically. To do this we draw a straight line AC from point (s_k, b_1) and (s_1, b_2) and a straight line BD from point (s_1, b_1) and (s_2, b_2). Their intersection gives a point (s_η, b_η) (fig.3). Since the $s_\eta = f_2(b)$ curve on the interesting us section is always very steep, and s_k is very close to s_η, the process of improving the values s and b converges rapidly. Practically it is sufficient to solve equations (2) and (6) two times each. Consequently the finding of s_η and b_η requires small time layouts.

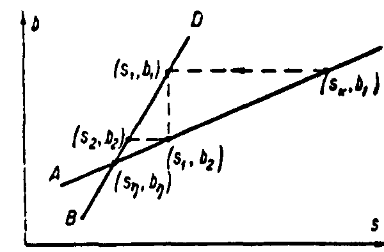

Fig.3. Determination of optimum values s_η and b_η by four points.

It is evident from formula (5(that with the rise in electrical conductivity and square of velocity of metal flow as well as with a reduction in frequency critical

and optical slippages decrease. Consequently for light metals and alloys of sodium type, sodium-potassium of high electrical conductivity, for which at a metallic channel may be assumed a rate of flow of 10-12 m/sec, the value s_{η} lies within limits of 0.2 - 0.35. For heavy metals with low electrical conductivity (bismuth, lead) the permissible rate of flow of which does not exceed 3-5 m/sec, optimum slipping increases to 0.7 - 0.8.

Determining the half width of the channel and pole graduation after finding optimum values s and b is done easily by formulas (1.7) and (1.8). Preliminary value of active pump length l' is fixed from ratio (III,3) with consideration of formulas (1.2) and (III,4). After determining the hydraulic losses in diffusors the value l is defined and the number of pole pairs is found:

$$p = \frac{l}{2\tau}. \tag{7}$$

The latter should be an integral number. That is why l and γ change somewhat to fulfill this condition.

Optimum ratios by minimum weight conditions of all active materials.

In appendix II were obtained expressions for the weight of active materials of an induction pump-copper and steel:

$$G_{Cu} = C_1 \frac{[\Delta^2 (1-s)^4 + K^2 (bs + n)^2][b + g_2 (1-s)]}{s(1-s)b}; \tag{8}$$

$$G_{Fe} = C_2 \frac{\sqrt{\Delta^2 (1-s)^4 + K^2 (bs + n)^2}}{s(1-s)b}. \tag{9}$$

In these expressions can be considered variable only s and b. The optimum values s_G and b_G corresponding to minimum weight of the active materials, would be desired to find, investigating the minimum of function

$$G(b, s) = G_{Cu} + G_{Fe}, \tag{9a}$$

One such an approach leads to highly difficult equations, the solution of which represents insurmountable difficulties. It therefore appears advisable to investigate each one of the functions (8) and (9) individually, and then to determine the optimum

values s_G and b_G. It can easily be seen, that tho function $G_{Fe}(s,b)$ has a minimum only at a change in s. When variating b from zero to infinity G_{Fe} decreaces monotonously from infinity to a certain constant value.

We will find at first the values of slipping, at which a minimum of copper and steel weight takes place. Taking the s derivative from expression (8) and equating it to zero, we will obtain an equation

$$\begin{gathered}(1+s)[K^2s^2-(1-s)^4]b^3+\\+[4nK^2s^2+g_2K^2(1-s)s^2-2d(1+s)(1-s)^4-\\-g_2(1+2s)(1-s^5)]b^2+[2g_2nK^2(1-s)s^2-\\-n^2K^2(1-3s)-d^2(1+s)(1-s)^4-\\-2g_2d(1+2s)(1-s)^5]b-\\-[g_2n^2K^2(1-2s)(1-s)+\\+g_2d^2(1+2s)(1-s)^5]=0.\end{gathered} \qquad (10)$$

Executing such an operation over dependence (9), we obtain:

$$\begin{gathered}(1+s)[K^2s^3-(1-s)^4]b^2-[n(1-3s)+\\+2d(1-s)^4]b-[n^2K^2(1-2s)+d^2(1-s)^4]=0.\end{gathered} \qquad (11)$$

Taking up the values s within limits of from 0 to 1 and solving equations (10) and (11) relative to b, it is possible to obtain slippage curves $s_{Cu} = \varphi_2(b)$ and $S_{Fe} = \varphi_2(b)$ corresponding to minimum weights of copper and steel. Each one of the s_{Cu} and s_{Fe} curves has asymptotes, corresponding to slippings s' and s". The first one of these can be found by approximation, assuming that the coefficient at b^3 in equation (10) is equal to zero. Since this coefficient is identical with the corresponding coefficient of equation (3), then s' is equal to s_k and is determined by formula (5). An accurate slip value s" is found from equation

$$K^2s^3-(1-s)^4=0. \qquad (12)$$

It can be noticed with ease, that s" exceeds somewhat s'. Consequently when solving equation (12) it is advisable to use the Horner table and the method of linear interpolations, remembering, that it is sufficient for us to find only one positive root, close to s'. It can be ascertained, that s_0 lies within narrow limits:

$$s_{Cu} < s_G < s_{Fe}. \qquad (13)$$

From inequality (13) is possible to change over to the following less accurate formula

$$s_к < s_G < s'' . \qquad (14)$$

which is suitable by the fact that the values s'' and s_k can easily be determined by expressions (5) and (12). We like to point out, that the boundaries of inequality (14) are practically sufficiently narrow. In practice it is practically everywheres necessary to abandon s_G, in order to obtain satisfactory efficiency. Consequently it can be assumed, that not an entirely strict inequality (14) appears to be perfectly applicable.

At selected electromagnetic loads and rate of flow of the metal the stepping away from s_G toward s_K leads to a reduction in the weight of the copper, rise in weight of steel and a rise in efficiency, and the shift in direction S'' involves opposite results.

Optimum in weight slipping s_G is always greater than s_η. From formula (5) and equation (12) is evident, that s_G, as well as s_η, is determined mainly by the parameter K. It depends little upon the pressure and delivery of the pump, and upon the electromagnetic loads as well.

After selecting slipping s_G optimum half height of the channel b_G it is advisable to determine by direct plotting of graphs $G_{Cu}(b)$, $G_{Fe}(b)$ and $G(b)$ by formulas (8) and (9). Such a method is less cumbersome, than the search for b_G by analytically investigating the minimum of function $G(b)$. A study of the minimum dependence $G_{Cu}(b)$ offers very small values b having no practical value, and the function $G_{Fe}(b)$ has no minimum. Since the value b_G should be selected with consideration of efficiency then it can also be found by the maximum efficiency conditions, because these values are very close. An analysis of the problem shows, that the optimum conditions for maximum efficiency and minimum weight require the selection of a channel width ratio to its height in the interval from 20 to 40.

<u>Appendix 1</u>. Derivation of Equation of Relative Losses. Pressure developed by the pump, equals:

$$p_{\text{н}} = \frac{B_m^2 \sigma v \delta l k_{oc}}{2(1-s)k_0} - \Delta p, \qquad (1.1)$$

where B_m - amplitude value of induction in the center of the liquid metal zone;

k_{oc}, k_o - coefficient of pressure reduction on account of lateral and longitudinal boundary effects [9, 12, 13];

Δ_p - hydraulic losses along active length of pump and in diffusors.

Hydraulic losses along the active length of pump Δp_1 at turbulent flow, as we as for smooth pipes, are determined approximately in the zone of Reynolds numbers $3 \cdot 10^3 < Re < 10^5$ by the Darsy-Weisbach formula with consideration of the empirical Blasius law:

$$\Delta p_1 = 2{,}8 \cdot 10^{-2} \nu^{0{,}25} \rho v^{1{,}75} b^{-1{,}25} l. \qquad (1.2)$$

Power losses in liquid metal P and in the walls of the channel P_k are expressed as:

$$P = \frac{2B_m^2 \sigma v^2 s^2 a b l k_0' k_{oc}}{(1-s)^2}; \qquad 1.3 \qquad\qquad P_\text{к} = \frac{2B_m^2 \sigma_\text{к} v^2 a b_\text{к} l k_0' k_{oc}}{(1-s)^2}. \qquad (1.4)$$

where k'_o - coefficient, considering losses from fields, due to longitudinal marginal effect [9].

The phase current in the inductor windings can be presented by the dependence

$$I = \frac{B_m v \sigma l}{\sqrt{2} m w k_w} \times$$

$$\times \sqrt{\left(\frac{k_\delta \omega}{\mu_0 \sigma v^2}\right)^2 \Delta^2 (1-s)^2 + \left(\frac{k_{oc} k_0'}{k_n} \frac{bs+n}{1-s}\right)^2}. \qquad (1.5)$$

where $n = \frac{\sigma_K b_K}{\sigma}$;

$k_\delta = k'_\delta k''_\delta k'''_\delta$ - resultant coefficient of gap; k'_δ -coefficient of serration (Carter); k''_δ - coefficient considering the rise in n.s.=no power due to the fact that the field with respect to the greater gap 2Δ does not appear to be plane parallel [12]; k'''_δ - coefficient of rise in n.s. as results of longitudinal boundary effect; k_n - coefficient determining the degree of field weakening in the center of the gap in comparison with the field on the surface of the inductor [12].

m, w, k_w-number of phases, number of in series connected turns in the phase and the winding coefficient of inductor winding.

Expression (1.5) is obtained if the magnetizing current component I is expressed by n.s., and the active one - through power losses, determinable by formulas (1.3) and (1.4) and EMF of the inductor.

Active resistance of inductor phase is determined by formula

$$r = \frac{4mw_1^2}{A\sigma_{\text{м}} l}\left(k_1 k_{rп}\tau + 2k_2 k_{rл} a\right). \qquad (1.6)$$

where σM - conductivity of winding copper at given temperature; $k_1 = \frac{l_л}{\tau}$ - ratio of length of frontal part of semicoil to pole graduation; k_2 = 1.04 - 1.15 - ratio of structural width to the width of the channel; k_{rp}, k_{rl} - coefficients of current expulsion in the slotted and frontal sections of the winding.

For a double layer winding k_1 = 1.5 - 1.7.

We will substitute in equation (1.6) instead of a and τ their values:

$$a = \frac{Q}{4vb}; \qquad 1.7 \qquad\qquad \tau = \frac{v}{2f(1-s)}. \qquad (1.8)$$

After transformation we will obtain:

$$r = \frac{2k_1 k_{rп} m w^2 f c}{A\sigma_{\text{м}} f l}\,\frac{b + g_1(1-s)}{b(1-s)}, \qquad (1.9)$$

where

$$g_1 = \frac{k_2 k_{rл}}{k_1 k_{rп}}\,\frac{fQ}{v^2}. \qquad (1.9a)$$

Power losses in inductor windings are determined by the expression

$$P_{\text{м}} \quad m r I^2. \qquad (1.10)$$

Using dependences (1.1), (1.3), (1.4) and (1.10) for finding the relative electrical losses:

$$\frac{\Sigma P}{P_2} = \frac{P + P_{\text{к}} + P_{\text{м}}}{P_{\text{н}} Q} \qquad (1.10a)$$

Disregarding the hydraulic losses, after transformation we will obtain an equation of relative electric losses in an induction pump:

$$\frac{\Sigma P}{P_2} = k_0 k_0' \left\{ \frac{s}{1-s} + \frac{n}{b(1-s)s} + \frac{1}{c}\,\frac{[b + g_1(1-s)]\,[\Delta^2(1-s)^2 + K^2(bs+n)^2]}{b(1-s)^2 s} \right\}.$$

where

$$c = k_1 \lambda^2 \frac{k_n^2 k_\omega^2}{k_{\kappa} k_o' \ldots_n} \frac{e_\kappa \ldots}{2 \ldots j};$$

$$K = \frac{k_{oc} k_o'}{k_n} \frac{\mu_o \ldots^2}{k_\omega \ldots} \ldots . \quad (I.11a)$$

Appendix II. Coefficients of equation (6)

$$a_0 = 2g_1 \Delta^2;\ a_1 = -(9g_1 + b)\Delta^2;\ a_2 = 3(5g_1 + b)\Delta^2;$$
$$a_3 = -[2(5g_1 + b)\Delta^2 + (c + 2g_1 nK^2)b + \ldots(g_1 - b)b^2];$$
$$a_4 = (c + g_1 nK^2)b + K^2(b + g_1 + 4n)b^2 -$$
$$- 2(cn + g_1 n^2 K^2) - 2b\Delta^2;$$
$$a_5 = 3[(g_1 + b)\Delta^2 + (cn + g_1 n^2 K^2) + n^2 K^2 b];$$
$$a_6 = -\frac{a_5}{3};\ \Delta = b + d;\ d = b_\kappa + b_i. \quad (I.11b)$$

Appendix III. Derivation of formulas for the weight of active materials. We will express the weight of active pump materials in the function of s and b. The weight of winding copper of two inductors equals

$$G_{Cu} = 4m(2ak_2 + k_1\tau)wq_\phi\gamma_{Cu}, \quad (III.1)$$

where q_ϕ - profile of winding copper of one phase; gamma$_{Cu}$ - specific weight of copper.

Using formulas (1.7) and (1.8) and expression

$$q_\phi = \frac{I}{j} = \frac{Al}{2mwj} \quad (III.1a)$$

after transformations we will obtain

$$G_{Cu} = \frac{A\tau\gamma_{Cu}k_1 l}{2jj}\left(\frac{g_2}{b} + \frac{1}{1 - s}\right) \quad (III.2)$$

where

$$g_2 = \frac{k_2 f \ldots}{k_1 \ldots^2}. \quad (III.2a)$$

The active length of the inductor will be found from expression (I.1) considering approximately the hydraulic pressure losses as constant:

$$l = \frac{2k_o(1 - s)(p_n + \Delta p)}{B_m^2 \ldots k_{oc}}. \quad (III.3)$$

Maximum induction value is obtained from formula (1.5) with consideration of expression for lineer load:

$$B_m = \frac{\mu_0 k_\omega A(1 - s)v}{\sqrt{2}k_\delta\omega\sqrt{\Delta^2(1 - s)^2 + K^2(bs + n)^2}} \quad (III.4)$$

Substituting expressions (III.3) and (III.4) in formula (III.2), we obtain a final expression for the weight of winding copper:

$$G_{Cu} = C_1 \frac{[\lambda^2(1-s)^2 + K^2(bs+a)^2][b + k_2(1-s)]}{s(1-s)b} \qquad (III.5)$$

where

$$C_1 = \frac{2\pi k_n k_1 k_\delta^2 (p_a + \Delta p)\omega\gamma_{Cu}}{k_{oc} k_w^2 k_0^2 \sigma^2 \tau \Delta j} \qquad (III.5a)$$

Weight of yokes of two inductors equals:

$$G_j = 4h_j a k_2 l \varphi_{Fe}\gamma_{Fe} \qquad (III.6)$$

where φ_{Fe} - coefficient of filling the package with steel; γ_{fe} - specific weight of steel; h_j - height of inductor yoke. Height of inductor yoke:

$$h_j = \frac{\tau B_{m0}}{\pi\varphi_{Fe}B_{mj}} = \frac{B_m k_n k_\delta''' v}{\varphi_{Fe}B_{mj}\omega(1-s)} \cdot \qquad III.7$$

Here B_{m0} and B_{mj} - amplitude values of induction on the surface and in the inductor yoke.

The weight of the serrations of two inductors is determined by the dependence

$$G_z = 4h_n a k_2 l \varphi_{Fe}\frac{b_z}{t_1}\gamma_{Fe} \qquad (III.8)$$

where h_p - height of slot; b_z- width of tooth; t_1 - tooth pitch.

The height of the slot h_p can be expressed . . through pole gradutaion and the coefficient $\lambda = \frac{h_p}{\tau}$, which can be acceptes as approximately constant, and the width of the tooth b_z equals:

$$b_z = \frac{t_1 B_{m0}}{\varphi_{Fe}B_{mz}} = \frac{t_1 B_m k_n k_\delta'''}{\varphi_{Fe}B_{mz}}, \qquad (III.9)$$

where B_{mz}- maximum value of induction in the tooth.

Taking into consideration expressions(III.6) - (III-9) the weight of inductor steel can be presented so:

$$G_{Fe} = G_j + G_z = \frac{lQB_m\gamma_{Fe}k_2k_nk_\delta'''}{\omega(1-s)b}\left(\frac{1}{B_{mj}} + \frac{\pi\lambda}{B_{mz}}\right). \qquad (III.9a)$$

Substituting in this formula the expressions (III.3) and (III.4) we will finally find

$$G_{Fe} = C_2 \sqrt{\frac{\Delta^2 (1-s)^4 + K^2 (bs+n)^2}{s(1-s)b}}, \qquad (III.10)$$

where

$$C_2 = \frac{2\sqrt{2}\, k_0 k_2 k_n k_\delta''' \, Q (p_н + \Delta p) \gamma_{Fe}}{k_{oc} k_w \mu_0 v^2 s A} \times \left[\frac{1}{B_{mj}} + \frac{\pi\lambda}{B_{mz}} \right]. \qquad (III.10a)$$

Appendix IV. Example 1. We will determine the optimum values b_η and s_η by the efficiency maximum for a UIN-1 type pump, designed by the Institute of Physics of the Academy of Sciences Latvian SSR fo bismuth at 400°C [14]. The given values: $p_n = 5.05 \cdot 10^5$ n/m², $Q = 2.08 \cdot 10^{-3}$ m³/sec, $f = 50$ c., $\sigma = 0.745 \cdot 10^6$ 1 ohm .m. Selected values: $b_k = 2 \cdot 10^{-3}$ m, $\sigma_k = 10^6$ 1/ohm.m, $b_1 = 0$, $v = 3.38$ m/sec, $j = 3.96 \cdot 10^6$ a/m², $d = b_k$; $\sigma_M = 22.3$ 1/ohm.m (at 400°C); $A = 7.45 \cdot 10^4$ a/m.

We will take the values of coefficients , which have been accepted in the given construction: $k'_\delta = 1.2$; $k''_\delta = 1.01$; $k'''_\delta = 1.0$; $k_\delta = 1.2 \cdot 1.01 \cdot 1.0 = 1.21$; $k_n = 1.01$; $k'_o = 1.15$; $k_o = 1.1$; $k_{oc} = 0.91$; $k_w = 0.966$; $k_1 = 1.5$; $k_2 = 1.1$; $k_{rp} = 1.1$, $k_{gl} = 1.0$. We calculate the dimensionless parameter of the pump and critical slip

$$K = \frac{0,91 \cdot 1,15}{1,01} \cdot \frac{4\pi\, 10^{-7} \cdot 0,745 \cdot 10^6 \cdot 3,38^2}{1,21 \cdot 314} = 2,91 \cdot 10^{-2};$$

$$s_к = \frac{2 + K - \sqrt{(2+K)^2 - 4}}{2} = \frac{2,0291 - \sqrt{2,0291^2 - 4}}{2} = 0,8435. \qquad 10b$$

The coefficients included in equations (2) and (6):

$$n = \frac{\sigma_к b_к}{\sigma} = \frac{10^6 \cdot 2 \cdot 10^{-3}}{0,745 \cdot 10^6} = 2,685 \cdot 10^{-3}\ м;$$

$$g_1 = \frac{k_2 k_{rп}}{k_1 k_{гл}} \cdot \frac{jQ}{v^2} = \frac{1,1 \cdot 1,1}{1,5 \cdot 1,0} \times \frac{50 \cdot 2,08 \cdot 10^{-3}}{3,38^2} = 7,37 \cdot 10^{-3}\ м, \qquad 10c$$

$$c = g_1 K^2 \frac{k_n^2 k_w^2}{k_{oc} k'_o k_2 k_{rп}} \frac{\sigma_м A}{2\sigma j} =$$

$$= 7,37 \cdot 10^{-3} \cdot 2,91^2 \cdot 10^{-4} \frac{1,01^2 \cdot 0,966^2 \cdot 22,3 \cdot 7,45 \cdot 10^4}{0,91 \cdot 1,15 \cdot 1,1 \cdot 1,1 \cdot 2 \cdot 0,745 \cdot 10^6 \cdot 3,96 \cdot 10^6} = 1,324 \cdot 10^{-6}\ м^2.$$

Substituting the values of these coefficients and s_k in equation (2), we obtain an equation for the first approximation b_1:

$$b_1^3 + 3{,}7\,b_1^2 - 2{,}5 = 0, \qquad (10d)$$

from which we find b_1=5,18 mm. A substitution of this value in equation (6) gives:

$$s_1^6 - 4{,}83 s_1^5 + 8{,}54 s_1^4 - 5{,}71 s_1^3 - 0{,}703 s_1^2 + 2{,}565 s_1 - 0{,}855 = 0; \qquad (10e)$$

The use of the Horner scheme and linear interpolation allows to find a positive root of this equation, close to s_k. It equals s_1 = 0.758. In ananalogous manner we find by s_1 the value b_2 = 4.0 mm, and by b_2 tho value s_2 = 0.753. The plotting, made by fig.3 gives: b_{η}= 3.92 mm, s_{η}=0.752. The found results are close to the ones accepted in the project: b = 3.0 mm, s = 0.755.

Example 2. We will determine the optimum values S_G and b_G applicable to pump IN-7 designed by the Institute of Physics of the Academy of Sciences Latvian SSR per minimum weight for sodium at 400°C [10, 14]. Given values: $p_n = 4.91 \cdot 10^5$ n/m², Q = $6{,}95 \cdot 10^{-2}$ m³/sec, f = 50c, $\sigma = 4.95 \cdot 10^6$ 1/ohm.m. Selected values: $b_k = 1.10^{-3}$ m, $\sigma_k = 10^6$ 1/ohm.m, b_i=0, v = 11,25 m/sec; j = $7.24 \cdot 10^6$ a/m², B_{mj} = 0,72 v.sec/m², B_{m2}= 1.21 v.sec/m², A = 18,64 x 10^4 a/m.

We will take the value of the coefficients, accepted in the given project:

$$k_\delta' = 1{,}13;\; k_\delta'' = 1{,}01;\; k_\delta''' = 1{,}17;$$
$$k_\delta = 1{,}13 \cdot 1{,}01 \cdot 1{,}17 = 1{,}335;\; k_n = 1{,}026;\; k_o = 1{,}2;$$
$$k_o' = 1{,}3;\quad k_{oc} = 0{,}834;\; k_\omega = 0{,}934;\; k_1 = 1{,}73; \qquad (10f)$$
$$k_2 = 1{,}04;\; \lambda = \frac{h_k}{\tau} = 0{,}267.$$

We are assuming that $\Delta p = 0.116_{pn}$; then $p_n + \Delta p$ = 5,47 x 10^5 n/m².

We calculate the dimensionless parameter of the pump and sliding $s' = s_k$:

$$K = \frac{k_{oc} k_o'}{k_n} \frac{\mu_0 \sigma v^2}{k_\delta \omega} = \frac{0{,}834 \cdot 1{,}3}{1{,}026} \times$$
$$\times \frac{4\pi 10^{-7} \cdot 4{,}95 \cdot 10^6 \cdot 11{,}25^2}{1{,}335 \cdot 314} = 1{,}98; \qquad (10g)$$
$$s' = s_k = \frac{K + 2 - \sqrt{(K+2)^2 - 4}}{2} =$$
$$= \frac{3{,}98 - \sqrt{15{,}84 - 4}}{2} = 0{,}270.$$

Slipping s" is determined from equation (12) at K = 1,98:

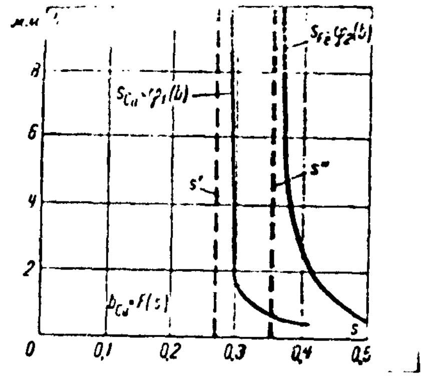

Fig.4.Graphs $S_{Cu} = \varphi_1(b)$ and $s_{Fe} = \varphi_2$ (b) and their asymptotes for IN-7 designed for minimum weight for pumping sodium.

We compare for this the equations of Horner table [11] and search for $s^* \searrow s_k$.

	1	−7,92	6,00	−4,00	1,00
0,354		0,354	−2,68	1,175	−1,00
	1	−7,566	3,32	−2,825	0

$$s^4 - 7{,}92s^3 + 6s^2 - 4s + 1 = 0. \quad (10h)$$

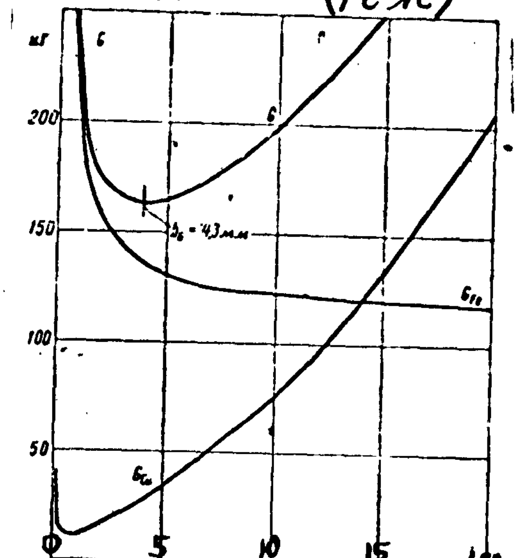

Fig.5.Dependences of copper weights and active steel weight and their sum upon the half-height of the channel for IN-7 pump at $s_G = 0.3$, $b_G = 4.3$ mm

It can be assumed, that $s^* = 0.354$ appears to be an approximated value of the root of the solved equation. Another one of its positive roots equals $s^* = 7.16$ and has no practical value as the third and fourth roots, which appear to be complex.

The optimum slipping value lies in the zone (fig.4)

$$s' = s_k = 0{,}27 < s_G < 0{,}354 = s''. \quad (10i)$$

We assume that $s_G = 0.3$. In theproject it was assumed that $s = 0.25$ for the purpose of raising the efficiency. To determine b_G we substitute $s_G = 0.3$ and the values of the coefficients in formulas (III.5) and (III.10). Dealing in various values b, we calculate G_{Cu}, G_{Fe} and G and plot graphs (fig.5) from which we find $b_G = 4.3$ mm. In the pump under question b = 10 mm, which led to a reduction in efficiency and to an increase in weight of construction.

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